

THE IMPACT OF THE THICKNESS OF THE POROUS MATERIAL ON THE PARALLEL PLATE CHANNEL FLOW OF REE-EYRING FLUID WHEN THE WALLS ARE PROVIDED WITH NON-ERODIBLE POROUS LINING

R. L. V. RENUKA DEVI¹, D. EBENEZER², V. NAGA RADHIKA³ & M. KRISHNA MURTHY⁴

¹Departmentt. of Mathematics, Sri Venkateswara University, Tirupati, Andhra Pradesh., India

²Departmentt. of Mathematics, Saveetha School of Engineering, Saveetha University, Tamil Nadu, India

³Department. of Mathematics, GITAM Deemed to be University, Bengaluru, Karnataka, India

⁴Department. of Mathematics, School of Applied Sciences, Reva University, Bengaluru, Karnataka, India

ABSTRACT

In this analysis the impact of the thickness of the permeable material on the parallel plate channel stream of Ree-Eyring liquid when the walls are furnished with non-erodible permeable lining is contemplated. The governing partial momentum equation is changed to ordinary differential equation by utilizing non-dimensional quantities and comprehended it analytically. The effects of governing parameters on the liquid velocity are appeared in graphically. We researched the stream in the free stream area and permeable stream districts by utilizing Darcy law and Ree-Eyring liquid model respectively.

KEYWORDS: *Ree-Eyring Liquid, Porous Lining & Parallel Plate Channel*

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INTRODUCTION

In recent years considerable interest has been displayed in the investigation of stream past permeable media due to its application in industrial, bio-physical and hydrological issues. In the investigation of stream past a permeable material it is standard to utilize the no-slip boundary condition at the permeable surface where the impact of porosity is dealt with by the continuity of the normal component of velocity. Some of the researchers are studied deformable porous channel in different channels (vertical, horizontal, inclination channel, slips) Beavers and Joseph [1], Krishna Murthy *et al.* [2-7], Sreenadh *et al.* [8], Eswara Rao *et al.* [9] and Krishna Murthy [10] some of the recent studies in Ree-Eyring fluids are given Ref. [11–14].

The present investigation manages the effect of the thickness of the permeable material on the parallel plate channel stream of Ree-Eyring liquid when the walls are given non-erodible permeable lining is considered. We examined the stream in the free stream area and permeable stream areas by utilizing Darcy law and Ree-Eyring liquid model separately. The governing velocity equation is solved by closed form solution. We found the flow in the free stream region and permeable stream regions by utilizing Darcy law and Ree-Eyring liquid model respectively.

Flow Geometry and Mathematical Model of The Problem

Consider, the steady flow of a Ree-Eyring liquid through a channel formed a channel by two rigid impermeable parallel plates at $y=0$ and $y=h$ is spoken in Figure 1. The lower wall is covered with a homogeneous and

isotropic permeable material of thickness $h' (\neq 0)$. Accordingly dividing the flow region into two zones, Zone 1 represents the region of the free stream between the upper impermeable wall and the nominal surface $y = h'$ and Zone 2 signifying the region of flow through the permeable material.

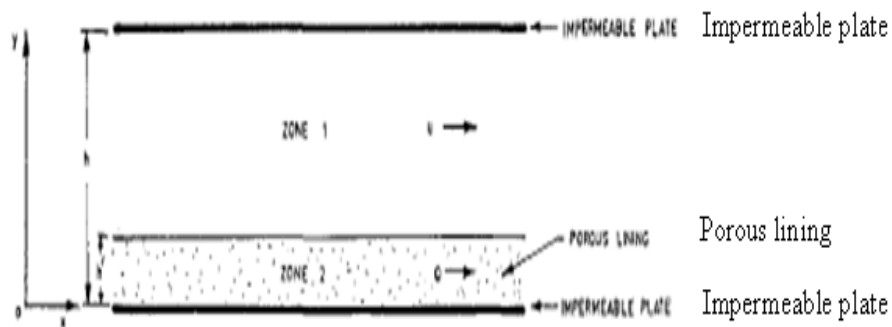


Figure 1: Physical Geometry of the Problem.

The stream which is caused by a uniform pressure gradient in the longitudinal direction in both the zones is thought to be fully developed and the liquid properties are altogether to be consistent. At the point the flow in Zone I is governed by the Navier-Stokes equation is

$$\mu \left(1 + \frac{1}{\mu_a \overline{B C}} \right) \frac{\partial^2 u}{\partial y^2} = \frac{\partial p}{\partial x} \quad (1)$$

and that in Zone II by the Darcy law

$$\mu \left(1 + \frac{1}{\mu_a \overline{B C}} \right) Q = -K \frac{\partial p}{\partial x} \quad (2)$$

The boundary conditions are as follows:

$$\left. \begin{aligned} u &= 0 \text{ at } y = h \\ \frac{\partial u}{\partial y} &= \frac{\alpha}{\sqrt{K}} (u_B - Q) \text{ at } y = h' \end{aligned} \right\} \quad (3)$$

The non-dimensional quantities are as follows:

$$v = \frac{u}{\bar{u}}, \eta = \frac{y}{h}, \xi = \frac{x}{h}, \pi = \frac{p}{\frac{1}{2} \rho \bar{u}^2}, R = \frac{\rho \bar{u} h}{\mu}, P = -\frac{R}{2} \frac{\partial \pi}{\partial \xi}, \sigma = \frac{h}{\sqrt{K}}, Q' = \frac{Q}{\bar{u}}, \varepsilon = \frac{h'}{h}, \beta = \frac{1}{\mu_a \overline{B C}} \quad (4)$$

Where u is the velocity, β is the Ree-Eyring fluid parameter, p is the pressure, μ is the dynamic viscosity, Q is the Darcy velocity, K is the absolute permeability of the material, u_B is the slip velocity at the nominal surface, α is the slip parameter, h is the height of the channel and h' is the thickness of the porous lining, ρ is the fluid density, R is the Reynolds number, \bar{u} is the average velocity in the channel, ε is the thickness of the porous channel.

From equations (1)–(4) we get the following equations are

$$\frac{d^2 v}{d\eta^2} = -\frac{P}{(1+\beta)} \quad (5)$$

$$Q' = \frac{P}{\sigma^2(1+\beta)} \quad (6)$$

The following non dimensional boundary conditions are

$$\left. \begin{aligned} v &= 0 \text{ at } \eta = 1 \\ \frac{dv}{d\eta} &= \alpha\sigma(v_B - Q') \text{ at } \eta = \varepsilon \end{aligned} \right\} \quad (7)$$

Where v_B is the slip velocity

SOLUTION OF THE PROBLEM

In this paper we solved the governing equation with the closed form solution. The solution of (5) satisfying (7) is

$$v(\eta) = (1-\eta) \left[\frac{P(1+\eta)}{2(1+\beta)} - \frac{P\varepsilon}{(1+\beta)} - \alpha\sigma v_B + \frac{\alpha P}{\sigma(1+\beta)} \right] \quad (8)$$

$$\text{Where } v_B = \frac{P(1-\varepsilon)[\sigma(1-\varepsilon)+2\alpha]}{2\sigma(1+\beta)[\sigma\alpha(1-\varepsilon)+1]}, \quad 0 < \varepsilon < 1 \quad (9)$$

We are enthusiasm to locate the quantitative impact of slip on the stream; we calculate the non-dimensional mass stream rate

$$M = M_1 + M_2 \quad (10)$$

$$\text{Where } M_1 = \int_{\varepsilon}^1 v d\eta = \frac{P}{12} \frac{(1-\varepsilon)^3}{(1+\beta)} \left[\frac{4 + \alpha\sigma(1-\varepsilon) - 6\alpha^2}{1 + \alpha\sigma(1-\varepsilon)} \right] + \frac{\alpha P}{2\sigma} \frac{(1-\varepsilon)^3}{(1+\beta)} = \frac{P}{12} A + \frac{P}{2} B \quad (11)$$

$$\text{and } M_2 = Q'\varepsilon = \frac{P\varepsilon}{\sigma^2(1+\beta)} \quad (12)$$

In order to bring out the impact of permeable lining in the channel we compare M with the mass stream rate M^* in the channel in the absence of lining where

$$M^* = \int_0^1 v d\eta = \frac{P}{3(1+\beta)} \quad (13)$$

Then the ratio of the mass stream rate with and without permeable lining is given by

$$\frac{M}{M^*} = \frac{A}{4} + \frac{3B}{2} + \frac{3\varepsilon}{\sigma^2} \quad (14)$$

RESULTS AND DISCUSSIONS

In this paper we analyzed the effect of the thickness of the permeable material on the parallel plate channel stream of Ree-Eyring liquid when the walls are givne with non-erodible permeable lining. We examined the stream in the free stream

region and permeable flow regions by utilizing Darcy law and Ree-Eyring liquid model respectively. The governing equation is tackled with closed form solution. The impacts of governing parameters on the liquid velocity from equation (8) are shown with the assistance of diagram for the stream in a channel with one side permeable lining.

The impact of thickness of porous layer \mathcal{E} on the liquid velocity $v(\eta)$ is appeared in figure 2. We saw that the velocity reduces for higher estimations of thickness of permeable layer. The effects of slip parameter α and the permeability parameter σ on the liquid velocity $v(\eta)$ are shown in figures 3 and 4. We saw that the liquid velocity rotates with expanding slip parameter and permeability parameter. The effect of pressure gradient P on the liquid velocity $v(\eta)$ is delineating in Figure 5. We have seen that the liquid velocity improves for higher estimations of pressure gradient. From Figure 6 speak to the impact of Ree-Eyring parameter β on the liquid velocity $v(\eta)$. We uncover that the liquid velocity increments with expanding Ree-Eyring parameter. The ratio of mass flow rate M/M^* covering one side permeable lining suppresses for higher estimations of the permeability parameter σ . Further for higher estimations of slip parameter α rotates the ratio of mass stream rate is represented in figures 7 and 8.

CONCLUSIONS

In this paper the impact of the thickness of the permeable material on the parallel plate channel stream of Ree-Eyring liquid when the walls are furnished with non-erodible permeable lining is contemplated. We researched the stream in the free stream region and permeable stream areas by utilizing Darcy law and Ree-Eyring liquid model respectively. The governing equation is explained with closed form solution. The targets of the present paper are as per the following:

- The liquid velocity diminishes with the impact of thickness of permeable layer, slip parameter and permeability parameter.
- The liquid velocity upgrades with an effect of pressure gradient and Ree-Eyring liquid parameter.
- The ratio of mass stream rate M/M^* covering one side permeable lining smoothers for higher estimations of the permeability parameter σ . Further for higher estimations of slip parameter α rotates.

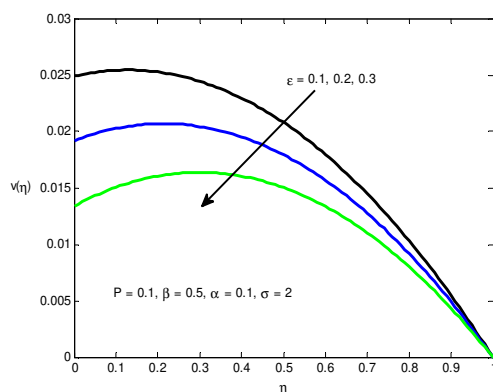


Figure 2: The Impact of \mathcal{E} on the Liquid Velocity $v(\eta)$.

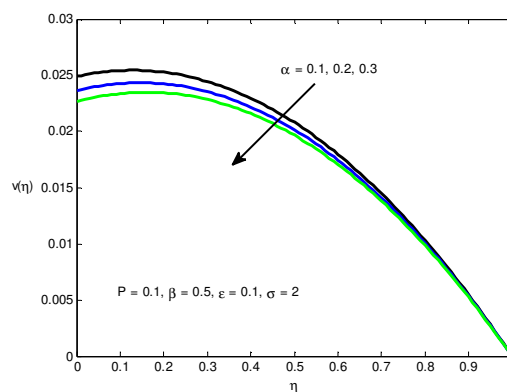


Figure 3: The Impact of α on the Liquid Velocity $v(\eta)$.

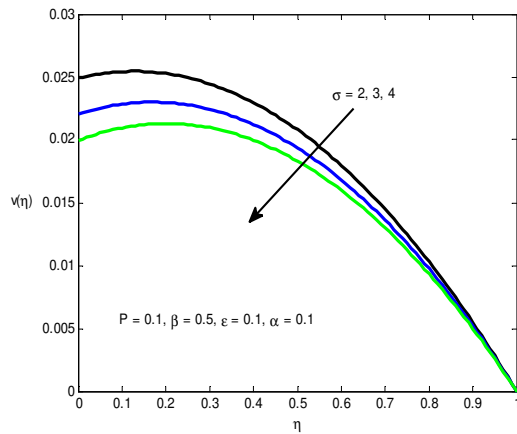


Figure 4: The Impact of σ on the Liquid Velocity $v(\eta)$.

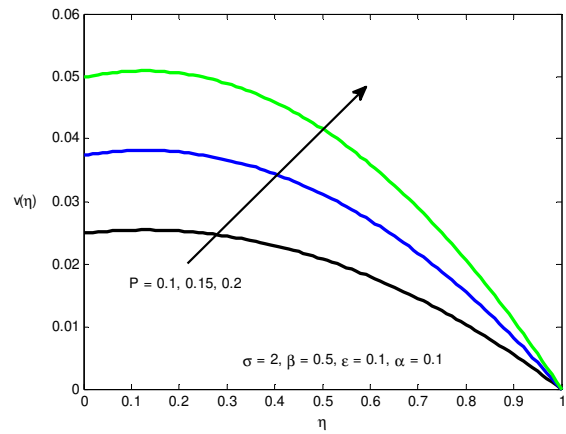


Figure 5: The Impact of P on the Liquid Velocity $v(\eta)$.

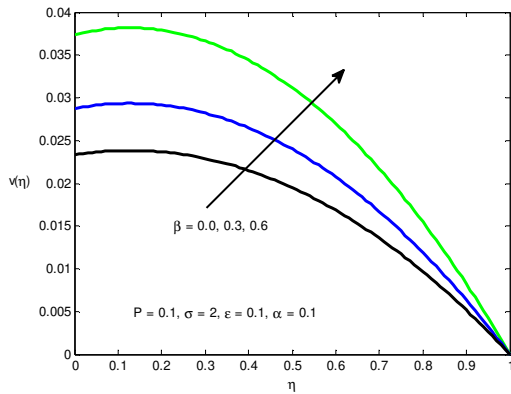


Figure 6: The Impact of β on the Liquid Velocity $v(\eta)$.

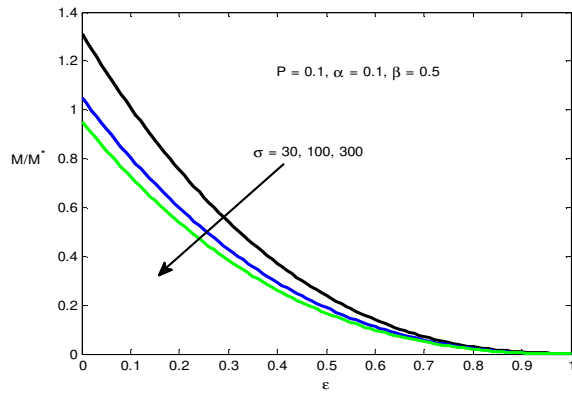


Figure 7: The Impact of σ on the Mass Stream Rate M/M^* for $\alpha = 0.1$.

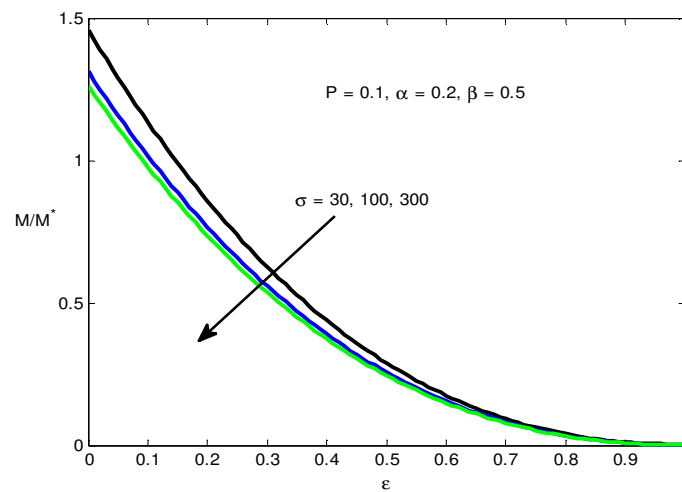


Figure 8: The Impact of σ on the Mass Stream Rate M/M^* for $\alpha = 0.2$.

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AUTHOR PROFILE



Dr. R.L.V. Renuka Devi has did her M.Sc; M.Phil and Ph.D. in Department of Mathematics, S. V. University, Tirupati, Andhra Pradesh, India and also working as an academic consultant in same University. Her research interest if

Fluid Dynamics. She has life membership in Indian Mathematical Society and Indian Science Congress Association. She has 9 research papers in reputed International Journals.



Dr. D. Ebenezer is presently serving as Assistant Professor in the division of Mathematics of Science and Humanities of Swetha School of Engineering, Saveetha University, Chennai, India. He has put up 44 years experience in teaching Mathematics to the UG and PG studies in various Govt. Colleges in Tamilnadu, India. His field of specialization is stochastic modeling operation research and digital signal process. At present he is actively doing research and collaborating with the group who are pursuing in computational Fluid dynamics field research.



Dr. V. Nagaradhika is working as Assistant Professor in GITAM University in the department of Mathematics. she completed my Ph.D., in fluid dynamics she have a credit of publishing 13 research papers in various National and International Journals (2 SCI and 1 Scopus) and also presented my research findings in five National, 4 International Conference. More so, she participated in five Faculty Development Programs and attended five Workshops hosted at various Universities. I delivered so many Guest Lecturers. She also hold a life time Membership in Andhra Pradesh Society for mathematics (APSFM).



Dr. M. Krishna Murthy is working as Assistant Professor, School of Applied Sciences, holds Ph.D. Degree in Mathematics, M.Phil degree (Mathematics), M.Sc. degree (Mathematics) from S.V. University. He has 3 years of teaching experience and 5 years of research experience. He has published 24 International journal papers in highly reputed journals. He has working as an Editorial Board Member for Composite Material Research Journal.

